CSCI 210: Computer Architecture
Lecture 17: Arithmetic Logic Unit

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- Born in 1924 in Egypt
- Invented the MOSFET (metal-oxide semiconductor field-effect transistor) with Dawon Kahng in 1960
- First truly compact transistor
- MOS transistors are the fundamental building blocks of today's electronics
- Most manufactured device in history
  - 13 sextillion MOS transistors manufactured as of 2018
- Went on to start a cybersecurity company, invented the "Atalla box" which secures most ATMs

Arithmetic and Logical Unit (ALU)

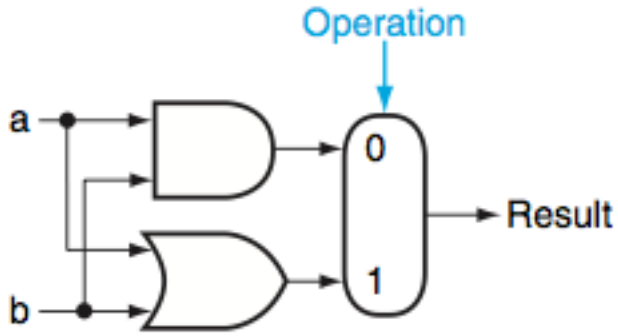
- Need to use digital logic to build a unit that can do basic computation – math, logical operations, etc.

- Needs to be 32 bits wide, since MIPS has 32 bit words.
  - Build out of 1-bit ALUs

Our ALU will support the following instructions:

- Or/Ori
- And/Andi
- Add/Addi
- Sub
- Nor/Nori
- Nand/Nandi
- Set less than

1-bit ALU: AND and OR



- Inputs go to both AND and OR
- Multiplexer selects AND or OR function for output

1-bit Binary Adding

0 + 0 = 0

0 + 1 = 1

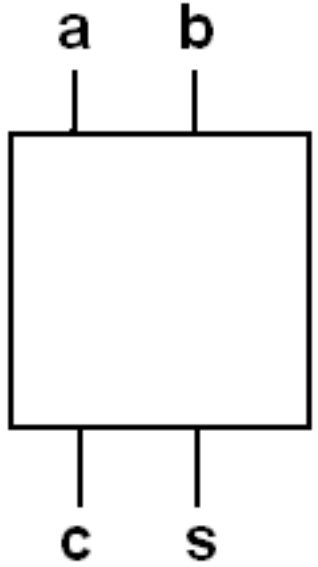
1 + 0 = 1

1 + 1 = 10

Need to account for two output bits!

Half Adder

- Inputs a, b
- Outputs sum and carry out
- Sum is the 1-bit result of adding a and b
- Carry out is the carry in the normal sense



Below is the truth table for the SUM output of a half adder. What is the Boolean algebra function that will give us this truth table?

a	b	Sum
0	0	0
0	1	1
1	0	1
1	1	0

A. a OR b

B. a XOR b

C. a AND b

D. a NOR b

E. None of the above

Below is the truth table for the CARRY output of a half adder. What is the Boolean algebra function that will give us this truth table?

a	b	Carry out
0	0	0
0	1	0
1	0	0
1	1	1

A. a OR b

B. a XOR b

C. a AND b

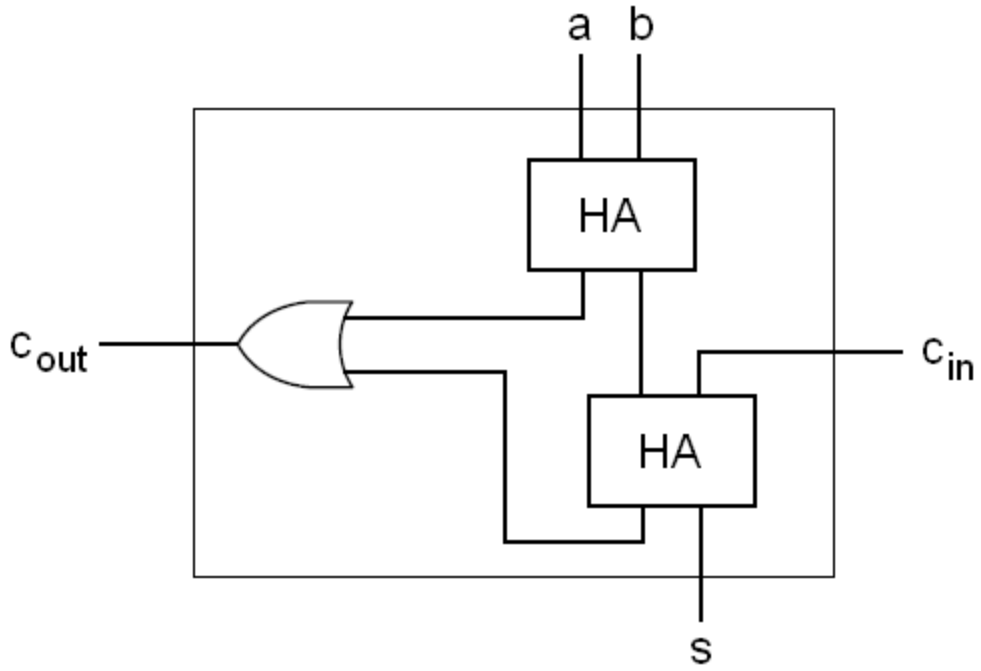
D. a NOR b

E. None of the above

Binary Addition with Arbitrary Number of Bits

- Just like regular, grade school addition
  - Make sure we carry a 1 to the next digit when needed
- Now we need to be able to account for the carry-in from the next least-significant bit
- Example: 7+5

Full Adder from Half Adders



- Need carry-in, as well as carry-out

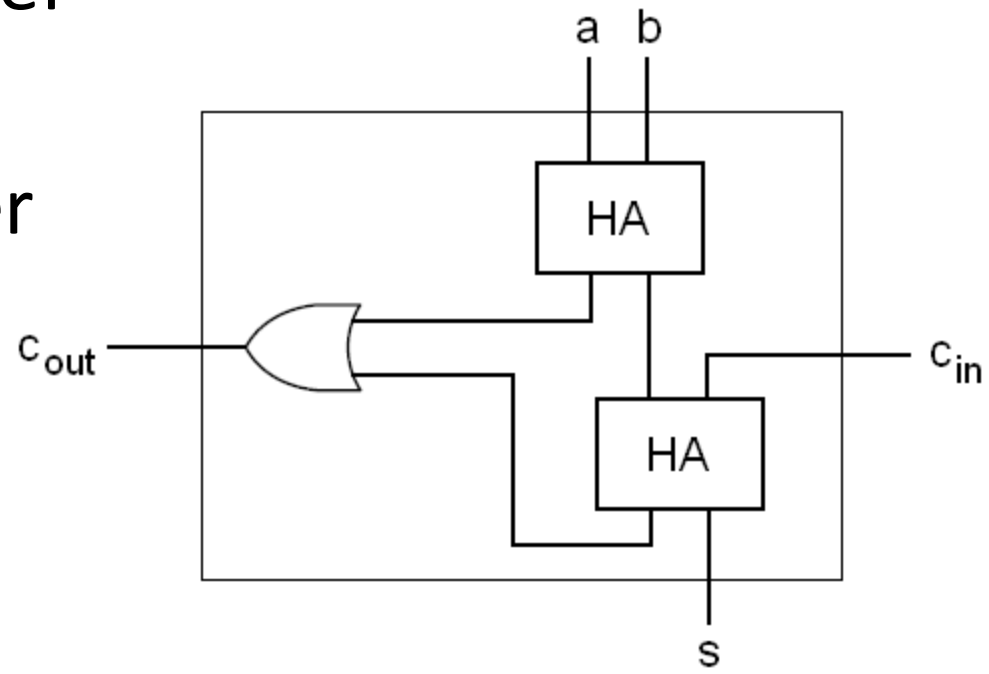
What if both half adders have carry-out?

A. We will get the wrong answer

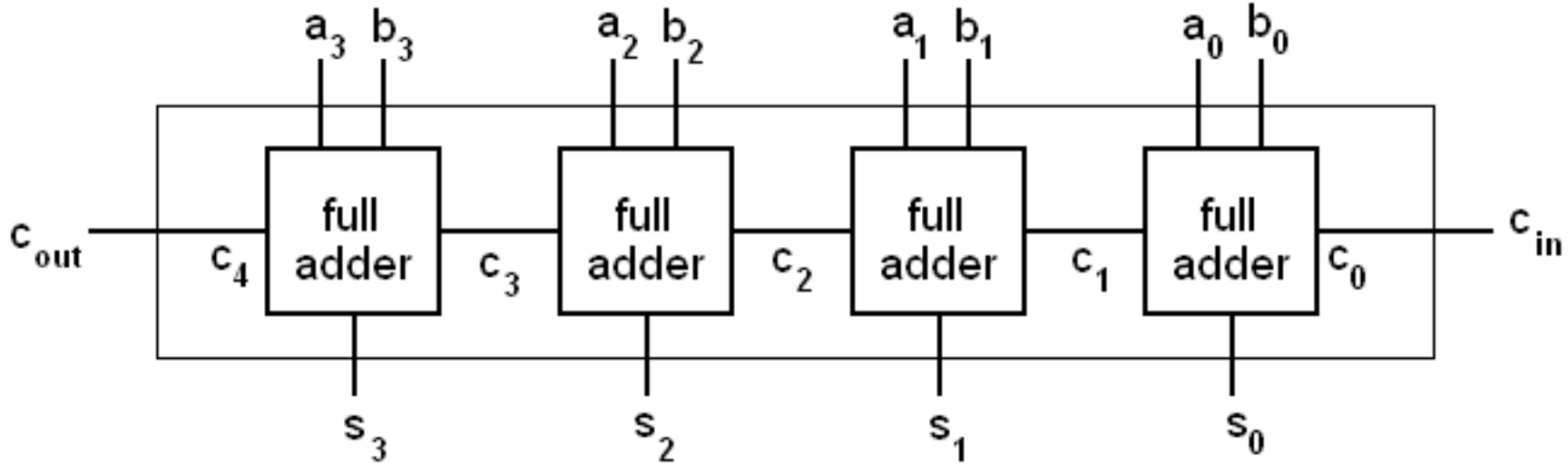
B. We will ignore it; the answer will still be correct

C. That will never happen

D. None of the above



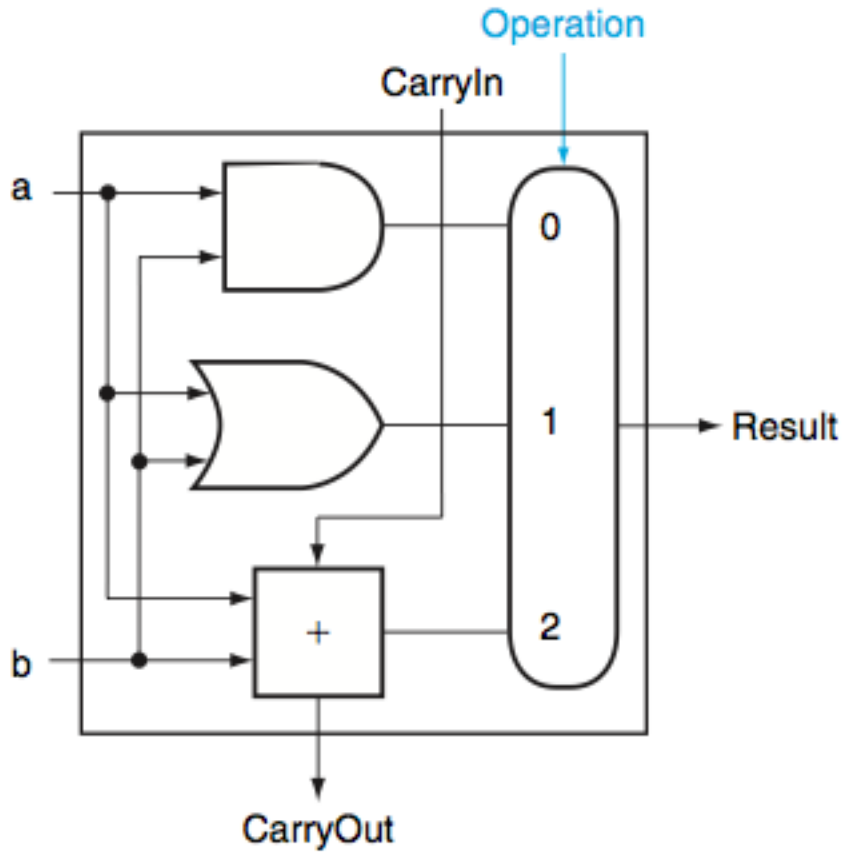
Ripple-Carry Adder



- Create adder for an arbitrary number of bits simply by connecting carry-out from adder n-1 to the carry-in for adder n

- Carry bit “ripples” up

1-bit ALU



Subtraction: a − b

- Just add negative version of b!

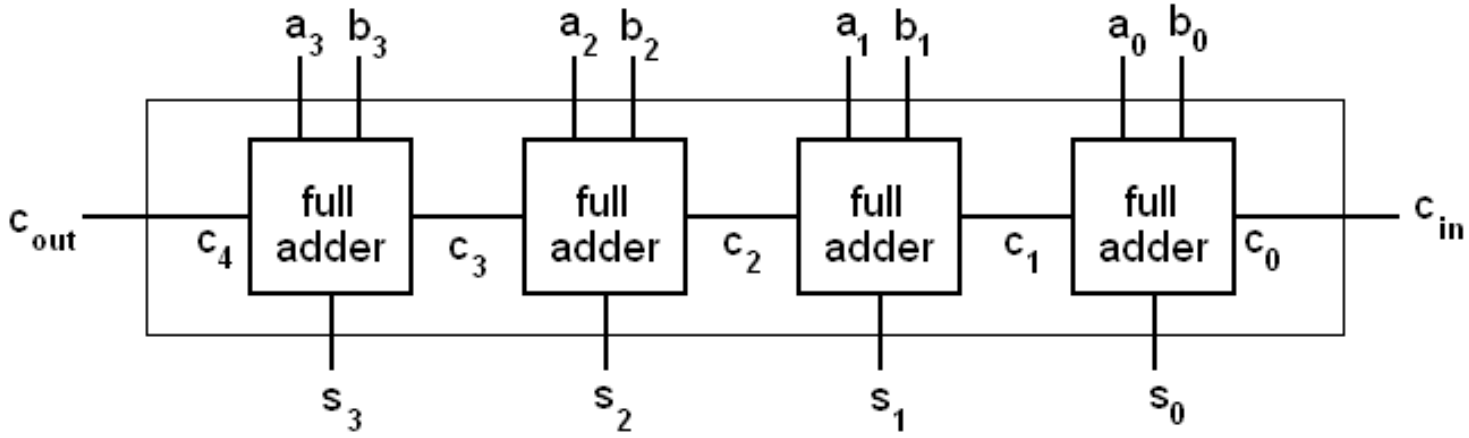
- To negate operand, transform to two's compliment
  - Invert each bit
  - Add one

We can use a NOT gate to invert the input. To add one to the input, we should

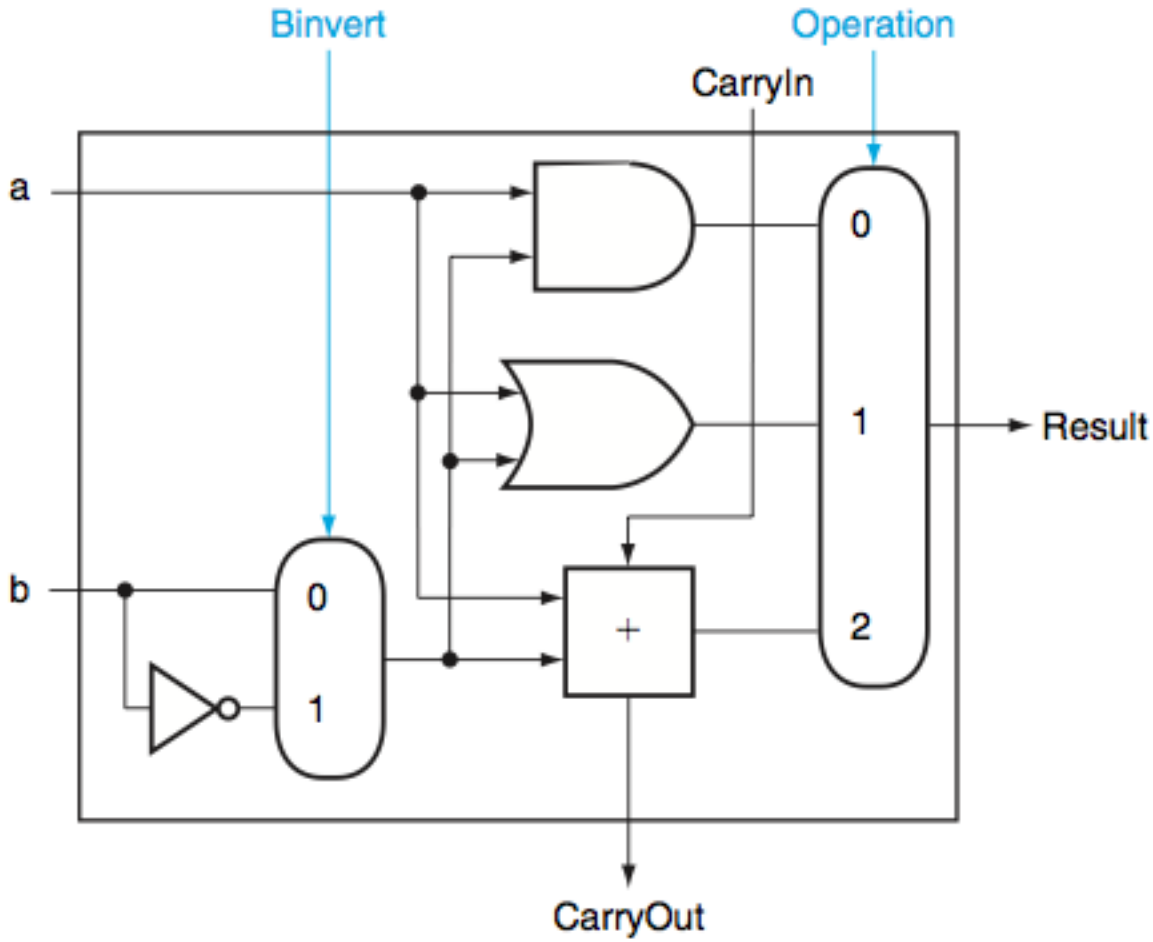
A. Set the carry-in for the least significant bit to 1.

B. Add a new “subtract” input that we set to 1 for subtraction.

C. Do something else.



1-bit ALU with Subtraction



Adding NOR

- Want to add NOR functionality

- DeMorgan's Law
  - $\overline{(A+B)} = \bar{A}\,\bar{B}$

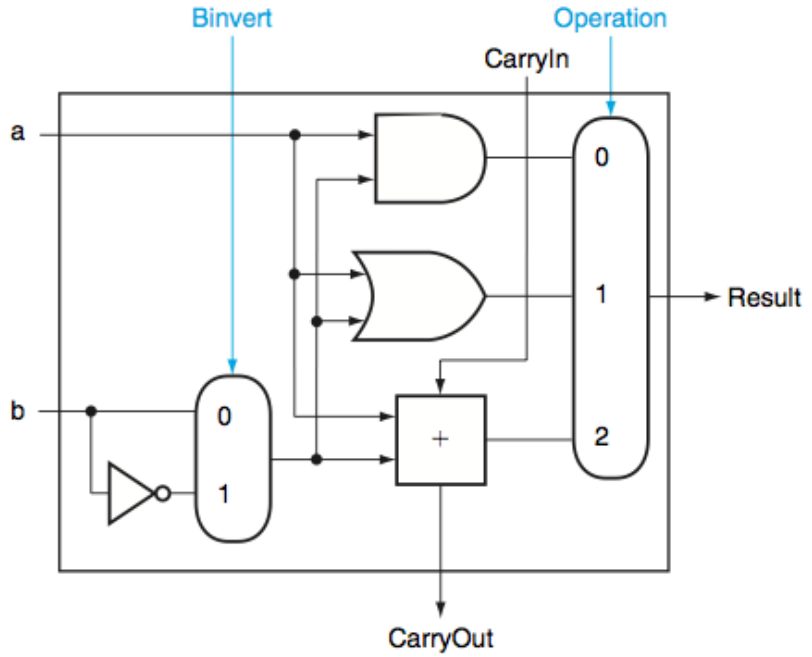
To add NOR to the ALU, we need to add

A. Nothing

B. The ability to invert A

C. A NOR gate

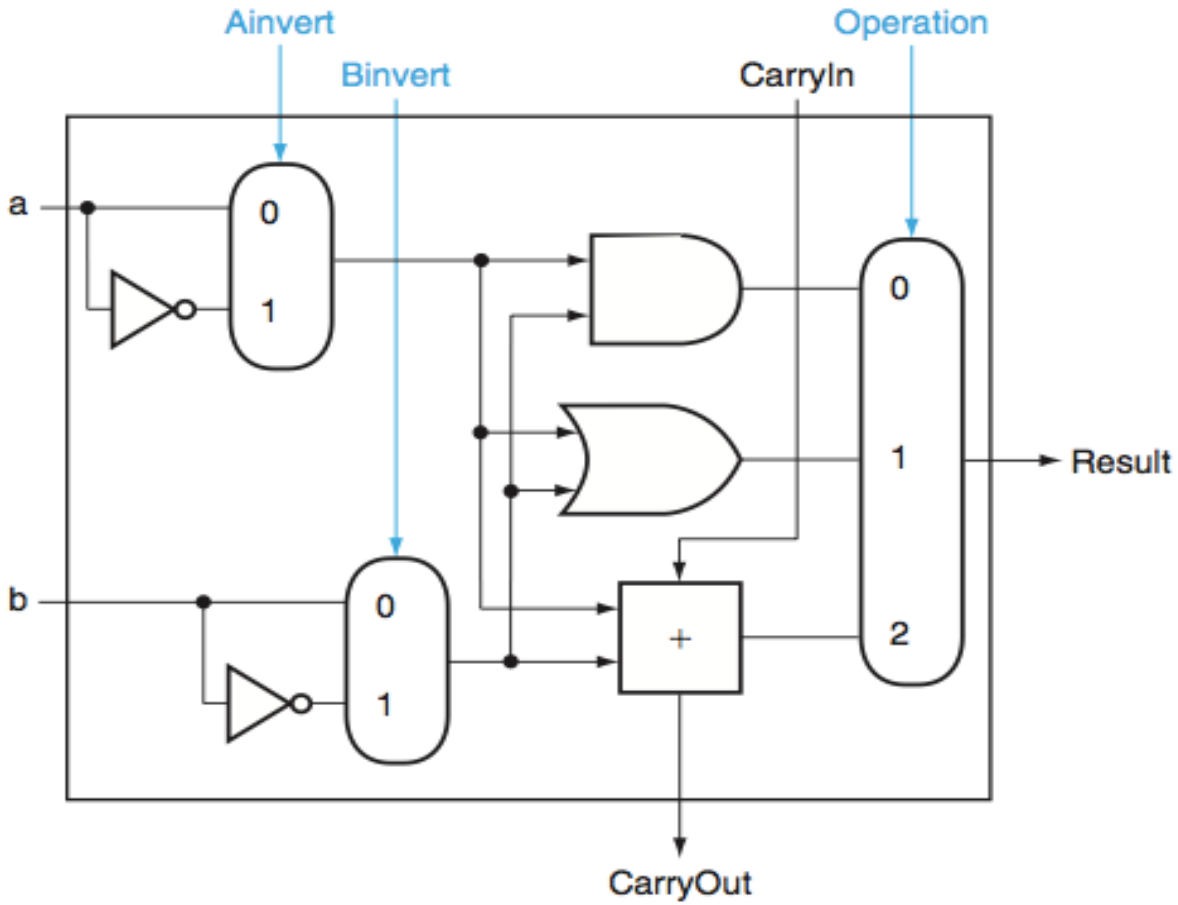
D. Something else



DeMorgan's Law

$\overline{(A+B)} = \bar{A}\,\bar{B}$

1-bit ALU with NOR

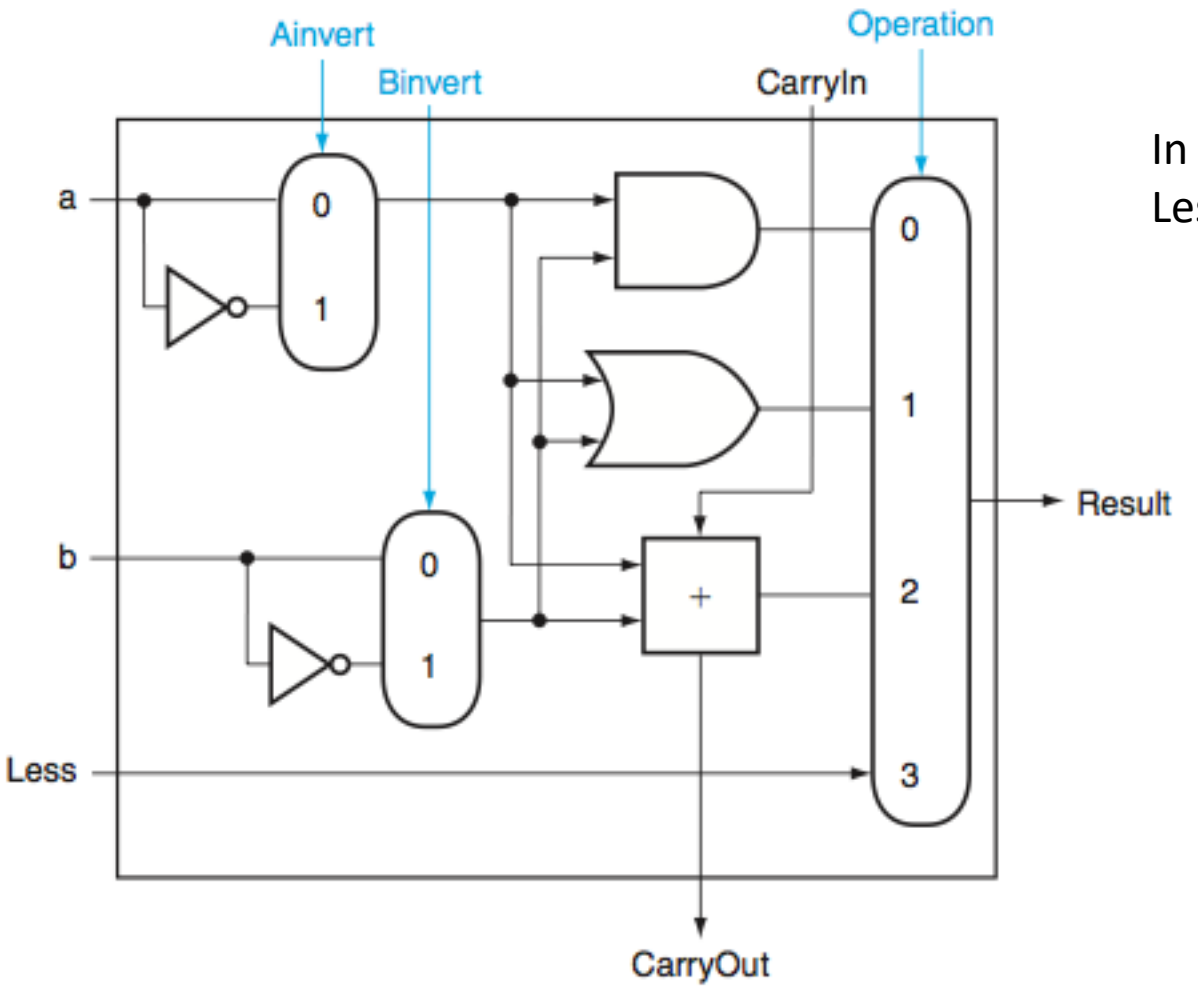


Adding slt

- `slt rd, rs, rt`
  - rd = 1 if rs < rt, and 0 otherwise

- Only sets least significant bit
  - All other bits are 0

1-bit ALU: Add new input for slt

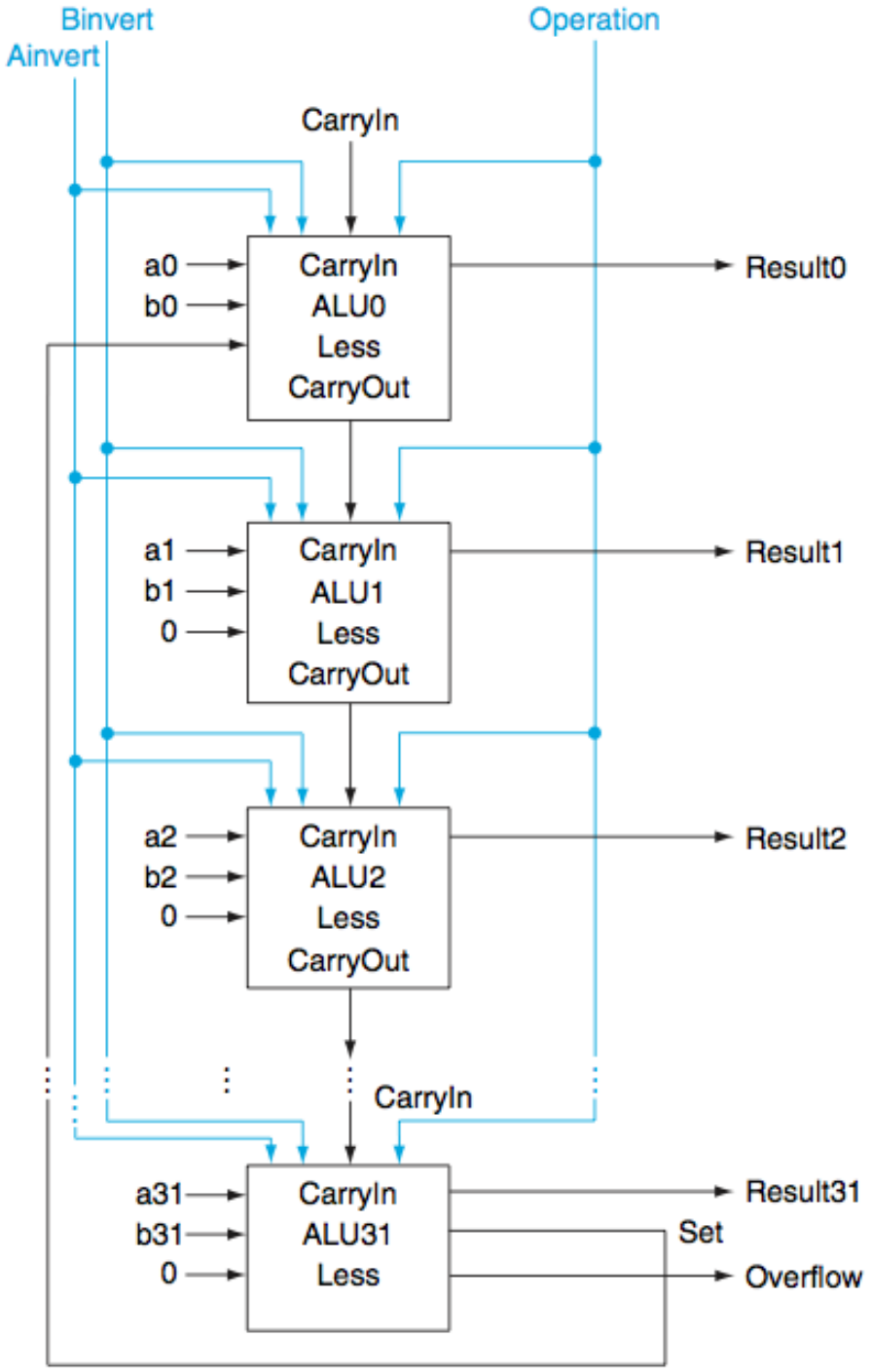


In all but the least significant bit,
Less will always be 0

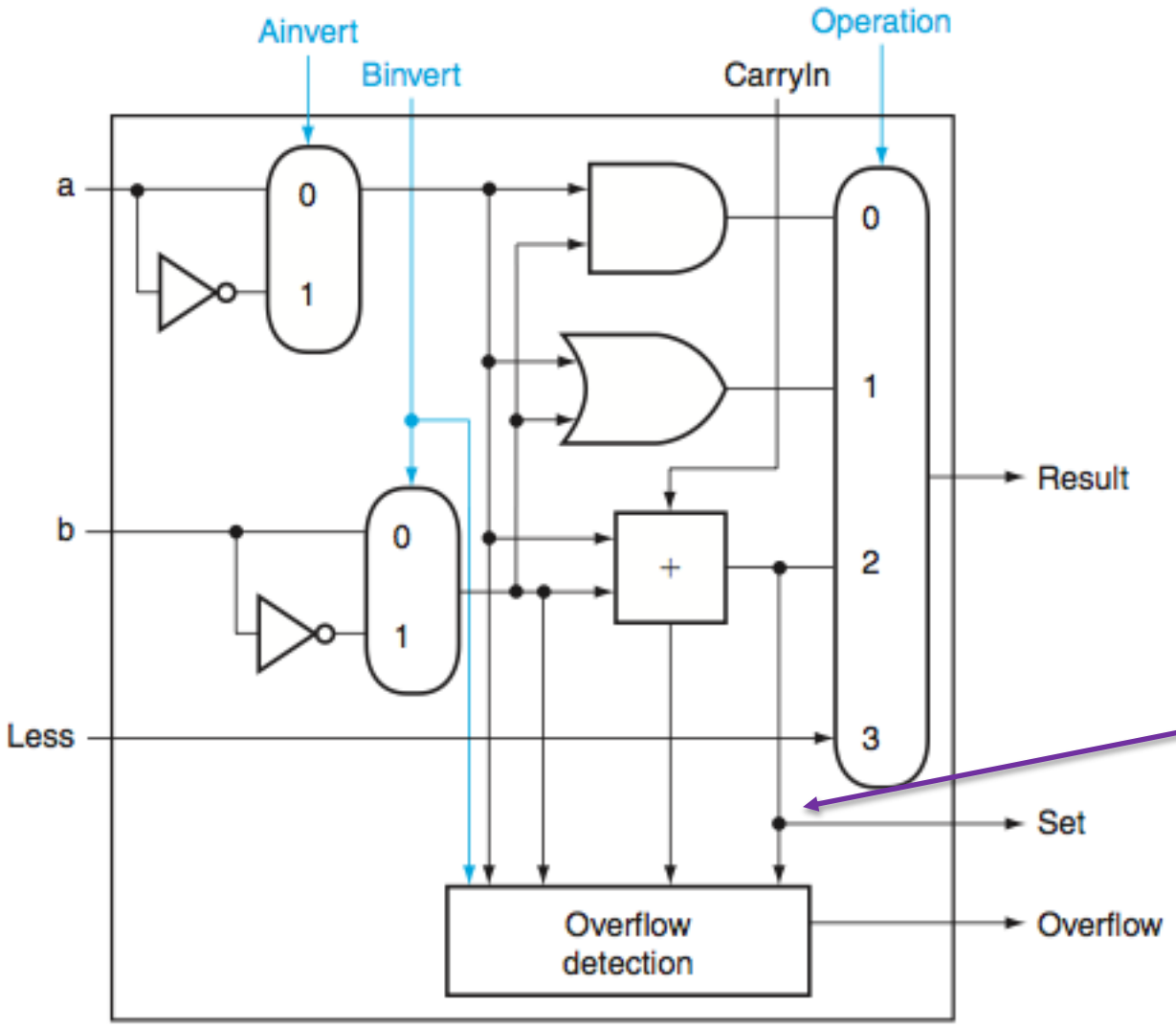
How do we tell if a < b?

- Subtract b from a
- If a – b < 0, then a < b
- We can check this by checking the most significant bit
  - MSB = 1, a < b

- Problem: Output is at Most Significant Bit, we need it at Least Significant Bit
- Solution: Special ALU for Most Significant Bit, with output for SLT
- Hook SET output into LESS input for Least Significant Bit



1-bit ALU for the Most Significant Bit



This doesn't always work! You'll fix it in problem set 6

Recall: Overflow

- If we add two n-bit numbers, we may end up with a n+1 bit number

- Hardware can detect this

a and b have different signs. Will adding them ever result in overflow?

A. Yes

B. No

Adding overflow detection to add

- If a and b have different MSBs, then there is no overflow
- If a and b have the same MSB, then
  - If the output MSB is different from the carryout, there is overflow

To check if the MSB is different from the carry out, check if

A. MSB AND Carry == 0

B. MSB OR CARRY == 1

C. MSB NOR CARRY == 0

D. MSB XOR CARRY == 1

E. None of the above

Reading

- Next lecture: Clocks, Latches and Flip flops
  - 3.6

- Problem set 5
  - Due Friday

- Lab 4
  - Due Monday